Port competition using capacity expansion and pricing

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Abstract

This study models two ports, serving the same hinterland, competing strategically using both pricing and capacity investment. Both ports are profit maximizers, and port expansions are lumpy, indivisible and irreversible. The decision making process of the two ports is analyzed using a two-stage game. In the first stage, two ports compete with each other on capacity expansion. In the second stage, they follow Bertrand competition with differentiated products conditional on realized port capacities. Within this formulation, we show the existence of unique Nash equilibrium in the pricing sub game, and the change of the equilibrium price with operation cost, market demand determinants and capacity sizes using comparative statics. In capacity investment game, we identified the pure strategy Nash equilibriums for different scenarios characterized by the incremental benefit of expansion and the annual capital cost of investment. Through both analytical study and numerical simulation, we show that the capacity expansion at any port will decrease the equilibrium prices at both ports, thus beneficial to the port users. Smaller port with more elastic demand and lower operation and investment cost is more likely to expand in an increasing market. Capacity expansion may result in lower total profit of the two ports, which is analogous to a Prisoner's Dilemma.

Keywords: Lumpy Capacity Investment, Pricing, Port Competition, Prisoners' Dilemma

1. Introduction

Container ports serving the common hinterland compete actively for global carriers. Due to the importance of container port activity to the local economy and the huge capital cost involved in port development and operation, the outcome of such competition not only has significant impact on the related private business, but also on the public sectors. Therefore, research on how ports can maintain their competitive edge and how this may affect the public and private sectors is of great importance.

Hong Kong is one of the leading container ports in the world, due to its unique position linking the fast economic development of mainland China to the outside world. However, in the past decade, its market share has been challenged by the container port development in Shenzhen, which not only generated huge capacity in a short period, but also enjoys many advantages, including lower operating costs and proximity to the hinterland. This can clearly be seen from figure 1, the container port throughputs from 1991 to 2008. It demonstrates that the throughput of Shenzhen increased from merely 51,000 TEUs to 21.4 million TEUs. This corresponds to an increase in market share from less than 1% to 47% for Shenzhen port, while that of Hong Kong decreased from 99% to 53%.

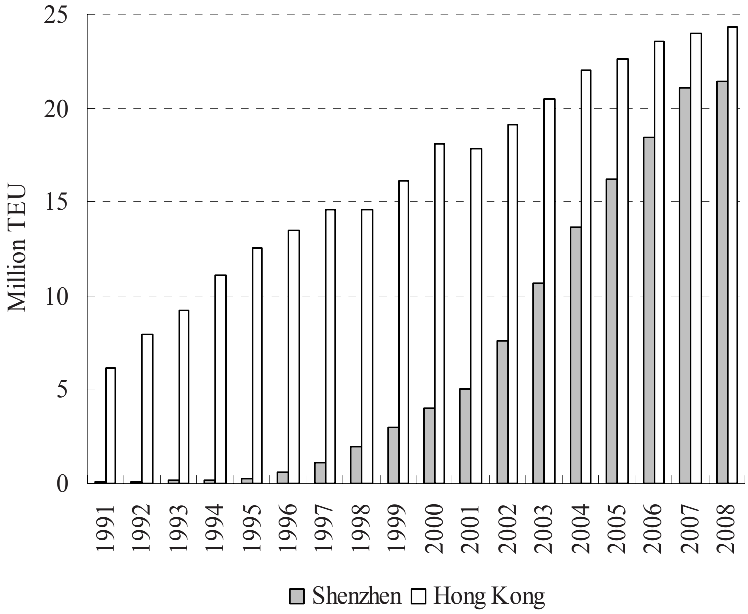

Figure 1: Container port throughputs of Hong Kong and Shenzhen port from 1991 to 2008
Source: Hong Kong Port Development Council, Shenzhen port information center, and UNCTAD

To maintain its competitive edge, Hong Kong Port can take many different strategies, which can be generally divided into two categories, short-term measures and long-term ones. *Short-term measures* include reducing price, increasing service quality, expediting import/export documentation processes, and other strategies that can reduce the user cost incurred when using the port. The *long-term strategies* include capacity expansion, which could improve container loading/unloading efficiency and reduce the vessel turn-around-time. While these measures all contribute to the attractiveness of a port, the competitative outcome is hard to predict when the two ports are competing strategically.

The purpose of this paper is to analyze the possible outcomes of port competition, taking port of Hong Kong and Shenzhen as an example. It models two ports, serving the same hinterland, competing strategically using both pricing and capacity investment. Both ports are profit maximizers, and port expansions are in lumpy, indivisible and irreversible. We use a two-stage model to analyze the decision making process. In the first stage, two ports compete with each other on capacity expansion. In the second stage, they follow Bertrand competition with differentiated products conditional on realized port capacities.

A literature review in capacity expansion and pricing reveals that there are many different research areas on this topic, from specific study in port, to general economics, game theory analysis, and operations research.

- In port study, most of the previous research on optimal port capacity and pricing is for single ports. For example, Devanney and Tan (1975) used dynamic programming to analyze optimal pricing and timing for capacity expansion in a monopoly port. Allahviranloo and Afandizadeh (2008) studied optimal investment on port development through minimizing the net present value of the total transportation cost, facility cost, dredging cost, operation cost, and benefit from the foreign shipping line at the national level. Literature on strategic port capacity investment and pricing is scarce.

- As to the general economics, Chenery (1952) studied the optimal capacity investment for exogenous demand increase over time, with economies of scale in plant size. Manne (1961, 1967) and Bean et al. (1992) analyzed capacity expansion with probabilistic growth, location and time. Abel and Eberly (1996) investigated capacity investment with reversibility issue. Starrett (1978) discussed optimal timing and size of the firm with depreciable capacity and increasing demand, from the perspective of welfare maximization. More recently, Demichelis and Tarola (2006) studied capacity expansion and dynamic monopoly pricing.

- With respect to game theory, Gilbert and Harris (1984) studied the competition between Nash competitors in indivisible and irreversible capacity investment. In their model, output is set equal to the capacity. Therefore, price and marginal cost are not an issue. Besanko and Doraszelski (2004) used dynamic programming method to study the capacity expansion in competitive market, and concluded that the capacity reversibility is a key determinant in firm size distribution in industry. Tabuchi (1994) developed a Hotelling model of spatial duopoly on two-dimensional space using two stage games. Firms select location in the first stage, and compete with each other using price at different location in the second stage. Some other relevant literature includes, but is not limited to, Gilbert and Lieberman (1987), Hay and Liu (1998) and Aguerrevere (2003).

- In operations research, Anupindi and Jiang (2008) discussed a duopoly model for production decision with capacity investment under demand uncertainty, competing in both price and quantity. Hall and Porteus (2000) developed a dynamic model in which firms compete by investing in capacity that affects the customer service level and consequently, the market share of each firm. Price is exogenous, and it has no effect on customer's preference. Liu et al. (2007) extend their work by incorporating a general demand form and further extend the game competition model to an infinite-horizon setting. Acemoglu et al. (2006) studied the capacity investment for service providers of a large-scale communication network and price competition, which is similar to our problem. However, in their research, cost of investment is continuous and proportional to the magnitude of the capacity. While in our problem, we consider lumpy capacity investment for port competition, i.e., each port will decide whether to invest or not, instead of the magnitude of the capacity expansion.

Compared with the existing literature, this paper has two unique properties. *First*, it considers two measures – capacity expansion and pricing – in an integrated framework of port competition. This is increasingly important because, facing an increasing market demand, port with large capacity can enjoy cost advantages from economy of scale, being more competitive in attracting global carriers and more likely to be successful in future competition. *Secondly*, it is the economics in port operation, rather than the port capacity constrain, that creates the needs for port expansion. Capacity is not binding. When port demand is higher than its capacity, a port can handle this with some extra cost, which implies a congestion cost. Therefore, it is the reduced congestion cost and possible revenue from the lower charge that made the expansion beneficial.

Next section presents model basics. Section 3 investigates the competition game between the two ports, with the strategic pricing in Section 3.1 and port expansion decision in Section 3.2. Section 4 provides a numerical analysis of competition between Hong Kong and Shenzhen container ports. A summary of the paper and the findings is in section 5.

2. Model Basics

This section presents the basic framework for the two-stage pricing – capacity expansion game. First, we adopt the Bertrand price competition with differentiated products (Baye and Kovenock, 2009). The base demand ($\bar{x}$) for cargo import and export services to and from the hinterland is assumed inelastic to the port price. The two ports serving this area have different initial market share α_k, where the index k (k=i or j) indicates individual port, and $\alpha_i+\alpha_j=1$. The two ports are perfect substitutes, and demand for one port decreases with its own price β_k, and increases with the price at the other port β_l,(l=i, j; $l \neq k$) i.e.,

$$x_i(p_i, p_j) = \alpha_i \bar{x} - \beta_i p_i + \beta_j p_j, \quad \text{and} \tag{1}$$

$$x_j(p_i, p_j) = \alpha_j \bar{x} - \beta_j p_j + \beta_i p_i \tag{2}$$

Secondly, we assume each port has its own operational cost function $V_k(x_k, C_k)$, with a positive marginal cost that increases with the throughput and decreases with capacity. This property enables the analysis for the benefits in capacity expansion - the reduced marginal operation cost. To simplify the mathematic derivation, we used a more specific functional form, $V_k(x_k, C_k)=f(C_k)x^2$, assuming $f_k(C)>0$ and $f_k'(C)<0$. The capacity of the port is not binding, reflecting in practice that a port can always handle more than its designed capacity with some additional cost. For numerical examples, we used $f_k(C_k)=\theta_k/C_k$, where θ_k is a positive constant. Larger θ_k means larger average marginal operational costs for a given capacity size.

Finally, we describe the two-stage game for strategic pricing and capacity expansion based on the real world decision-making process, as depicted in Figure 2. At the beginning of stage 1, each port decides whether to expand its capacity, knowing the capacity expansion behavior of the competitor and anticipating the pricing strategy of the two ports after this period. Each port can only add a fixed capacity at a time, and the incremental capacity is the same in two ports. Capacity, once added, is not removable. Then in stage 2, having observed the realized capacities at two ports, each port sets a price to maximize its profit within that period, conditional on the pricing strategy of the competitor.

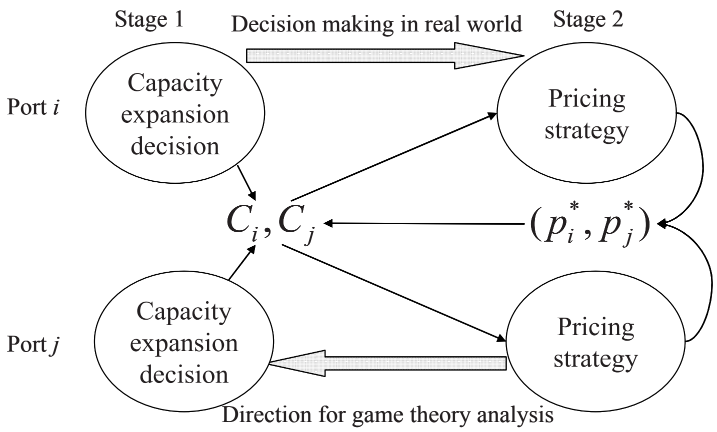

Figure 2: Illustration of the two-stage game

To analyze this decision process, we start from stage 2, where each port set its best price in response to the price of the competitor, and the available capacities at both ports. Then in stage 1, each port determines its best strategy in capacity development. At this stage, if a port chooses to develop, the new capacity will be C_k^1; otherwise, it will be C_k^0. Therefore, in the next section, we first present price competition in stage 2, followed by the capacity investment game in stage 1.

3. Capacity Expansion and Pricing Game

This section presents the game theory analysis for strategic decision making in capacity expansion and pricing. Following backward induction, we show first the pricing subgame in stage two, then the capacity expansion game in stage one. In price subgame, to begin with, we show the Nash equilibrium prices of the two ports for existing capacities. Following that, we present the static analysis of the equilibrium profit and price change with important parameters. In capacity expansion game, we analyze the pure strategy Nash equilibriums for capacity expansion.

3.1. Pricing Subgame

As introduced in the previous section, we adopt the Bertrand competition with differentiated products to model the strategic pricing behavior in the second stage. Under this specification, there exists a unique Bertrand equilibrium (Cheng, 1985), which states that port equilibrium price always exceeds its marginal cost, and two ports can have different prices and positive profits. We first characterize the mutual best response function of each port and show the equilibrium price. To link the pricing strategy in this stage with the capacity expansion outcomes in the first stage and other demand parameters, we apply comparative static analysis on equilibrium price and profit with respect to the concerned parameters. Assuming each port chooses the best price to maximize its profit based on the existing port capacity, i.e.,

$$\max_{p_k} \Pi_k(p_i, p_j, C_k) = p_k x_k(p_i, p_j) - f_k(C_k) x_k^2(p_i, p_j) \tag{3}$$

(the second order condition $\frac{\partial^2 \Pi_k(p_i, p_j, C_k)}{\partial p_k^2} = -2\beta_k - 2\beta_k^2 f(C) < 0$), then the price satisfying the first order condition

$$p_k - \frac{x_k(p_i, p_j)}{\beta_k} = 2 f_k(C_k) x_k(p_i, p_j) \tag{4}$$

maximizes the profit for port *k*. This is also the best response to the price of the other port. This first order condition can also be expressed in the elasticity term, $-\varepsilon_k^* = 1 + 2f_k(C_k)\beta_k$, where $\varepsilon_k^* < 0$ is the demand elasticity of port *k* at the optimal point. The right hand side of (4) is the marginal cost of port *k*, which is increasing in x_k due to congestion.
For each port, the mutual best response function can be obtained by substituting the corresponding demand function into its first order condition, and solving its own price p_k in terms of the price of the other port, p_l :

$$p_k(p_l) = \frac{[1 + 2\beta_k f_k(C_k)]\beta_l p_l + \alpha_k \bar{x}[1 + 2\beta_k f_k(C_k)]}{\beta_k[2 + 2\beta_k f_k(C_k)]}. \tag{5}$$

Using the elasticity term, (5) can also be written as $p_k(p_l) = \frac{\varepsilon_k^*}{\beta_k(\varepsilon_k^* - 1)}(\beta_l p_l + \alpha_k \bar{x})$. Replacing $\frac{\varepsilon_k^*}{\beta_k(\varepsilon_k^* - 1)}$ with φ_k^*, it can be further simplified as

$$p_k(p_l) = \varphi_k^*(\beta_l p_l + \alpha_k \bar{x}) \tag{6}$$

This can be seen in figure 3, which illustrates the best response pricing strategy for each port. According to Cheng (1985), there will be a Nash equilibrium price pair (p_i^*, p_j^*) at the intersection of these two best response curves for the given capacity level of each port.

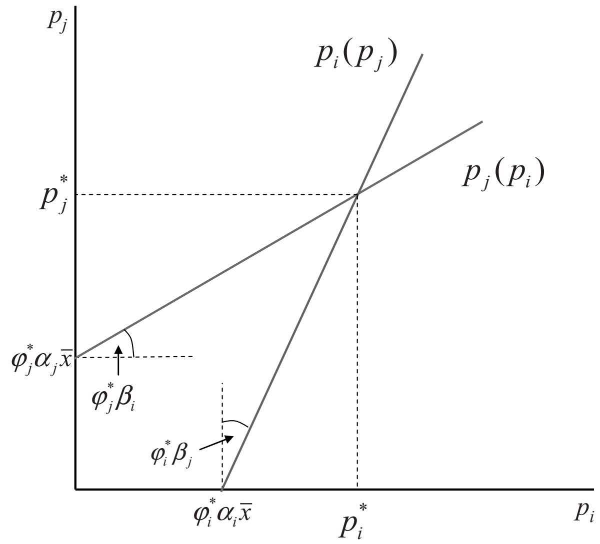

Figure 3: Mutual optimal price response functions for the two ports

3.1.1. Comparative Statics Analysis for Equilibrium Price and Profit

The purpose of comparative statics analysis is to exam the changes in equilibrium price, throughput, and profit with respect to the changes in the important parameters, such as the capacity, demand, and price sensitivity. The results from this analysis can be summarized as follows: Here we give a summary of the results.

(1) The equilibrium price (throughput) of one port increases with all the parameters that increase the demand of the port ($\bar{x}$, α_k, β_l), decreases with the parameters that decreases its demand (α_l, β_k).
(2) Capacity expansion in either port will decrease the equilibrium price. It will increase its own throughput, and decrease the throughput of the competitor.
(3) Port expansion decreases competitor's profit.
(4) Port expansion can have positive gain if $1+2f_k(C_k)\beta_k - 2f_l(C_l)\beta_l > 0$. Since the change of marginal cost *w.r.t.* own price is $\frac{\partial MC_k}{\partial p_k} = -2f_k(C_k)\beta_k$, the above condition can be written as $\frac{\partial MC_k}{\partial p_k} - \frac{\partial MC_l}{\partial p_l} < 1$. That is, if the difference in the change of marginal cost with respect to price change between port *k* and *l* is less than one, port *k* can increase its profit through expansion.

The Appendix contains the detailed mathematical derivations for the change of equilibrium price p_k^* and quantity x_k^* w.r.t. the change in demand parameters and capacity, as well as the change in equilibrium profit w.r.t. capacity change.

To summarize, capacity expansion can increase its throughput at the expense of the other port and decrease prices at both ports. This can reduce the profit at the non-expansion port. However, the profit at the expansion port may decrease or increase, depending on the relative sensitivity of marginal cost *w.r.t.* prices at two ports. For one unit decrease in price from capacity expansion, if the increase in marginal cost between the expansion port and the other port is less than one, port expansion will increase its profit.

The properties derived in this section have practical implications for optimal pricing strategy under competition. First, both ports could charge a higher price to cover the congestion cost when the market is good. Second, for larger port with higher existing market share, the optimal price could be higher. If demand is sensitive to this price, it is not optimal to charge higher price, because the user will shift to the other port.

Although capacity expansion can reduce congestion, it may not necessarily increase its profit. However, capacity expansion of one port will definitely reduce the profit of the other. Therefore, to make port expansion decision, it is necessary to consider the behavior in capacity expansion, in addition to the pricing of the competitor.

3.2. Capacity Expansion Game

This section explores the strategic investment behavior of the competing ports. In this game, each port decides whether to invest in capacity expansion, knowing that the other port is making the same decision. Denoting the equilibrium price, throughput, profit and annual capital cost in stage 2 for port k as $p_k^*(C_i, C_j)$, $x_k^*(C_i, C_j)$, $\Pi_k^*(C_i, C_j)$, and I_k, where C_i, C_j are the capacities for port i and port j respectively, for port i, facing the capacity of the other port C_j, its decision problem is:

$$\max_{C_i \in \{C_i^0, C_i^1\}} \Pi_i^*(C_i, C_j) - I_i \cdot 1_{C_i = C_i^1} \tag{7}$$
$$= \max_{C_i \in \{C_i^0, C_i^1\}} p_i^*(C_i, C_j) x_i^*(C_i, C_j) - V_i(x_i^*(C_i, C_j), C_i) - I_i \cdot 1_{C_i = C_i^1}.$$

If $I_i < \Pi_i^*(C_i^1, C_j) - \Pi_i^*(C_i^0, C_j)$, i.e., for the given capacity at port j, the gain from capacity expansion can offset the capital cost in that period, then the port should expand. Let

$$\Delta\Pi_i^*(C_j) = \Pi_i^*(C_i^1, C_j) - \Pi_i^*(C_i^0, C_j) \tag{8}$$
$$\Delta\Pi_j^*(C_i) = \Pi_j^*(C_i, C_j^1) - \Pi_j^*(C_i, C_j^0) \tag{9}$$

(8) and (9) are the gain in port expansion for given capacity of the competitor. Ports decide whether to expand by comparing these with the annual capital cost. The decisions are analyzed using a normal form game, such as the one in Table 1, which contains the corresponding net profits for the two ports. The letter *Y* and *N* stand for two possible development strategies – *Y* for expand, *N* for not expand. To see the decision-making process, we first investigate a special simpler case where the two ports are identical.

Table 1: Net Profits at Equilibrium price for Different Capacity Investment Decisions

	j:N	*j:Y*
i:N	$[\Pi_i^*(C_i^0, C_j^0); \Pi_j^*(C_i^0, C_j^0)]$	$[\Pi_i^*(C_i^0, C_j^1); \Pi_j^*(C_i^0, C_j^1) - I_j]$
i:Y	$[\Pi_i^*(C_i^1, C_j^0) - I_i; \Pi_j^*(C_i^1, C_j^0)]$	$[\Pi_i^*(C_i^1, C_j^1) - I_i; \Pi_j^*(C_i^1, C_j^1) - I_j]$

3.2.1. Identical Competitors

Here we assume the two ports have same demand and cost functions, and the same initial capacity before and after expansion, i.e., $C_i^0 = C_j^0 := C^0$ and $C_i^1 = C_j^1 := C^1$, and the costs are the same, i.e., $I_i = I_j := I$. Since the two ports are identical, we have $\Pi_i^*(C, C) = \Pi_j^*(C, C)$ ($C = C^0$ or C^1) and $\Pi_k^*(C^1, C^0) = \Pi_l^*(C^0, C^1)$. Using C^0 for other port not expand and C^1 for other expand, we have $\Delta\Pi_i^*(C) = \Delta\Pi_j^*(C) := \Delta\Pi^*(C)$, ($C \in \{C^0, C^1\}$). Under these assumptions, the investment decision rules are as follows:

(1) $I < \min(\Delta\Pi^*(C^0), \Delta\Pi^*(C^1))$: The annual capital cost is less than the gain, regardless of expansion decision of the other ports. In this case, both ports will choose to expand and (*Y, Y*) is the unique Nash equilibrium. For example, if both ports have serious congestion problems facing a rapid

increase in demand, they will both expand, irrespective of the decision of the other.

(2) $\Delta\Pi^*(C^1)<I<\Delta\Pi^*(C^0)$: The annual capital cost is less than the gain if the other port does not expand, and more than the gain if it does. For each port, if the other port expands, it will not expand since $I>\Delta\Pi^*(C^1)$. If the other port does not expand, it will expand because $I<\Delta\Pi^*(C^0)$. Therefore, it is optimal for the port to choose the opposite strategy as the competitor. (*N*, *Y*) and (*Y, N*) are two Nash equilibriums. This is similar to the situation where the demand is just enough for one port to develop. If both develop, they will end up over capacity, which is bad for both ports.

(3) $\Delta\Pi^*(C^0)<I<\Delta\Pi^*(C^1)$: The annual capital cost is more than the gain if the other port does not expand and less than the gain if it does. This happens when the expansion of one port exerts a detrimental impact on the other. It is better for the other to follow the strategy, to counteract the impact of the expanding port. Hence, one port better expands, if the other port expands, since $I<\Delta\Pi^*(C^1)$; if the other port does not expand, it will not expand since $I>\Delta\Pi^*(C^0)$. Thus (*N*, *N*) and (*Y*, *Y*) are two Nash equilibriums.

(4) $I>\max(\Delta\Pi^*(C^0), \Delta\Pi^*(C^1))$: The annual capital cost for expansion is larger than the gain regardless of the number of expanding ports. In this case, both ports will not expand. (*N*, *N*) is the unique equilibrium of the investment decision for the whole competition game.

3.2.2. General Competitors

Having explored the decision rules for the identical competitors, this section begins to investigate the capacity investment game where ports have different operational cost and market demand functions. First, for each port, we define a symbol for possible gains from expansion, with possible strategies of the other port.

Let :

$L_i = \Delta\Pi_i^*(C_j^0)$: The gain for port *i* when port *j* does not expand.

$M_i = \Delta\Pi_i^*(C_j^1)$: The gain for port *i* when port *j* expands.

$L_j = \Delta\Pi_j^*(C_i^0)$: The gain for port *j* when port *i* does not expand.

$M_j = \Delta\Pi_j^*(C_i^1)$: The gain for port *j* when port *i* expands.

Following the discussions for identical ports, for each port, we have the following scenarios and their corresponding decision rules:

- $I_k<\min(L_k, M_k)$: it is optimal to expand;
- $M_k<I_k<L_k$: it is optimal to make a different decision from its competitor;
- $M_k>I_k>L_k$: it is optimal to make the same decision as its competitor;
- $I_k>\max(L_k, M_k)$: it is optimal not to expand.

When the competitors are not identical, $L_i\neq L_j$, $M_i\neq M_j$, there will be 16 scenario combinations, as for each decision rule in one port, there will be four responses from the other port. For each combination, two ports determine their respective best expansion strategy, using the normal form game presented in Table 1. Table 2 lists all the possible equilibrium strategies for each of the 16 combinations. Unlike the pricing subgame that has a unique equilibrium, the capacity investment game may have multiple equilibriums or no equilibrium in some scenarios. When there is no equilibrium, we cannot predict the strategy for the player with certainty. We analyze such scenarios in the following.

Table 2: Nash Equilibrium of the Capacity Investment Game

	$I_j<\min(L_j, M_j)$	$M_j<I_j<L_j$	$M_j>I_j>L_j$	$I_j>\max(L_j, M_j)$
$I_i<\min(L_i, M_i)$	(*Y, Y*)	(*Y, N*)	(*Y, Y*)	(*Y, N*)
$M_i<I_i<L_i$	(*N, Y*)	(*Y, N*)(*N, Y*)	No Equilibrium	(*Y, N*)
$M_i>I_i>L_i$	(*Y, Y*)	No Equilibrium	(*Y, Y*)(*N, N*)	(*N, N*)
$I_i>\max(L_i,M_i)$	(*N, Y*)	(*N, Y*)	(*N, N*)	(*N, N*)

When $M_k>I_k>L_k$, i.e., port expansion is only feasible when the other port also expands, there are two equilibriums (*Y*, *Y*) and (*N*, *N*). This situation occurs when the investment cost of a port is larger than the gain from the port expansion ($I_k>L_k$). However, if the competitor expands, it will exert serious impact on the target port. Thus, the best response for the target port is also to expand, to counter-balance the impact from the other port. However, if ports know the action of the other, no port will expand first, as $I_k>L_k$. Therefore, as a response strategy, (*Y, Y*) is impossible.

For the scenario $M_k<I_k<L_k$, there are two equilibriums (*Y*, *N*) and (*N*, *Y*). For each port *k*, $M_k<L_k$ means that the gain from expansion will be weakened by the expansion of the competitor, thus the port will be reluctant to make expansion decision if the other port expands. By comparing the net profits of each port under the two equilibriums, we find there is no dominating equilibrium.

For the scenarios $M_k < I_k < L_k$ and $L_l < I_l < M_l$, for first port *k*, it is optimal to make a different expansion decision than the second port *l*; while for port *l*, it is optimal to make the same decision as port *k*. Thus there is no equilibrium.

From Table 2, it is obvious that if one port has a clear indication on its strategy [$I_k<\min(L_k, M_k)$ or $I_k>\max(L_k, M_k)$], there will be a unique pure strategy Nash equilibrium. All the border cells in the table contain at least one port that has a very clear choice for expansion strategy. $I_k<\min(L_k, M_k)$ happens when the port expansion cost is low, and/or the gain from expansion is high. These are often associated with small scale ports and increasing demand in the hinterland, such as the port in Shenzhen in the past. On the other hand, $I_k>\max(L_k, M_k)$ often happens when the port is in an area with high construction costs, large scale operation, and the stable demand, which is much like the case for Hong Kong Port. If one port is in either one of the above conditions, the strategy of the other is easy to formulate, especially when it is depend on the strategy of the first one.

However, the investment cost – gain relation will change, and at certain stage, neither port can indicate a clear direction. In this case, the competition strategy will be more interactive and inter-dependent, such as the strategies in the middle cells of Table 2. At present, the scale of port capacity and port throughput in Shenzhen and Hong Kong is very close. However, there are still big differences in the port construction cost and operation cost between these two ports. Thus, port of Shenzhen can still give a clear indication on expansion strategy. If these costs increase in the future, then the port development strategies of the two ports will be more interdependent.

Next, we analyze how profit changes with capacity expansion. As the capacity expansion is discrete, we cannot analyze the gain using comparative static method. Therefore, we check the sign of the gain by subtracting the profit of non-expansion from that with expansion. Assuming the likelihood of expansion is proportional to the gain from expansion, we can determine which port is more prone to expand.

The proof of the nature of incremental profit with respect to its own capacity expansion is in Appendix. Note that the port expansion will increase its own profit only when $2\beta_i f_i(C^u)-2\beta_j f_j(C^l)+1>0$, where C^u is the new capacity that port *i* is considering reaching, and C^l is the existing capacity at the other port. In marginal cost, the condition is:

$$\frac{\partial MC_i^u}{\partial p_i}-\frac{\partial MC_j^l}{\partial p_j}<1 \qquad (10)$$

This condition states that for an expanding port to increase its profit, the marginal cost increasing rate should not be higher than that of the competitor by one. This points out that if the demand sensitivity is high at one port, expansion is not a good strategy to use as it cannot increase the profit.

3.2.3. Numerical Examples

In this section, we show the application of capacity expansion game using numerical examples, to illustrate pure Nash equilibriums in table 2.

Example 1 *In this example, we set* $\bar{x}=1$, α_i*=0.6,* α_j*=0.4,* β_i*=0.1,* β_j*=0.2,* θ_i*=2,* θ_j*=1,* C_i^0*=3,* C_j^0*=2,* ΔC=1. I_i*=0.014,* I_j*=0.01, where* ΔC *is the capacity increment due to the lumpy investment.*

The profits with different investment decisions under equilibrium prices are in Table 3. In this example, L_i=3.1240-3.1104=0.0136, M_i=3.0175-3.0032=0.0143, L_j=1.1751- 1.1638=0.0113, M_j=1.1478-1.1364=0.0114. This corresponds to the scenario $L_i<I_i<M_i$ and $I_j<\min(L_j, M_j)$. Since port j always has an incentive to expand, port i will expand, too. Therefore, the pure strategy Nash equilibrium is (Y, Y). Compared with the (N,N) decision pair, the (Y,Y) decision pair result in lower profits for both ports since 3.0175-0.014=3.0035<3.1104, 1.1478-0.01=1.1378 <1.1638. This is a Prisoners' Dilemma.

Table 3: Example 1 when investment cost not subtracted

	$j:N$	$j:Y$
$i:N$	3.1104; 1.1638	3.0032;1.1751-0.01
$i:Y$	3.1240-0.014;1.1364	3.0175-0.014;1.1478-0.01

The next four numerical examples are to show the impacts of base demand, market share, price sensitivities, and cost parameters on the investment decisions.

Example 2 *(Base demand and market share) In this example, we set* $\beta_i=\beta_j$*=0.1,* $\theta_i=\theta_j$*=1,* $C^0{}_i=C^0{}_j$*=2,* ΔC*=1 and* $I_i=I_j$*=0.05. Table 4 provides the equilibrium investment decisions for different base demand, and market shares of each port.*

Table 4: Impact of market demand on capacity expansion decision

	$\bar{x}=1$	$\bar{x}=2$	$\bar{x}=3$	$\bar{x}=4$
α_i=1;α_j=0	*(N, N)*	*(Y, N)*	*(Y, Y)*	*(Y, Y)*
α_i=1/2;α_j=1/2	*(N, N)*	*(Y, Y)*	*(Y, Y)*	*(Y, Y)*
α_i=1/3;α_j=2/3	*(N, N)*	*(N, Y)*	*(Y, Y)*	*(Y, Y)*

The results of example 2, given in Table 4, show that with the increase of the base demand $\bar{x}$, the two ports will be more likely to expand. When the total base demand is fixed at $\bar{x}=2$, the port will be more likely to expand when its market share increases, and the other port tends not to expand when its market share decrease.

Example 3 *(Price Sensitivity) In this example, we set* $\bar{x}$ *=1,* $\alpha_i=\alpha_j$=0.5, $\theta_i=\theta_j$*=1,* $C^0{}_i=C^0{}_j$*=2,* ΔC*=1 and* $I_i=I_j$*=0.0125. Table 5 shows the equilibrium investment decisions for different values of* β_i *and* β_j.

Table 5: Impact of price sensitivities on the expansion decisions

	β_j=0.1	β_j=0.2	β_j=0.3	β_j=0.4
β_i=0.1	*(Y, Y)*	*(Y, Y)*	*(N, Y)*	*(N, Y)*
β_i=0.2	*(Y, Y)*	*(Y, Y)(N, N)*	*(N, N)*	*(N, N)*
β_i=0.3	*(Y, N)*	*(N, N)*	*(N, N)*	*(N, N)*
β_i=0.4	*(Y, N)*	*(N, N)*	*(N, N)*	*(N, N)*

Example 3 shows the impact of price sensitivities on expansion decisions. When β_i=0.1, the increase in the value of β_j will make port i reluctant to expand. This is because the gain in port expansion decreases with the increase in cross price sensitivity. Table 5 also shows that the increase in the values of both β_i and β_j will make both ports not expand. When demand is insensitive to prices (small β_k), expansion can generate positive gain from the reduction in congestion. Otherwise, the gains from expansion cannot offset the increase in congestion cost due to the increased throughput.

Example 4 *(Operational Cost) Following example 3, set* $\bar{x}=1$, $\alpha_i=\alpha_j=0.5$, $\beta_i=\beta_j=0.1$, $C_i^0=C_j^0=2$, $\Delta C=1$ *and* $I_i=I_j=0.015$. *The investment decisions at equilibrium are in Table 6 for different values of* θ_i *and* θ_j.

Table 6: Impact of operational cost on the expansion decisions

	θ_j=0.5	θ_j=1.0	θ_j=1.5	θ_j=2.0
θ_i=0.5	(*N, N*)	(*N, N*)	(*N, Y*)	(*N, Y*)
θ_i=1.0	(*N, N*)	(*N, N*)	(*N, Y*)	(*N, Y*)
θ_i=1.5	(*Y, N*)	(*Y, N*)	(*Y, Y*)	(*Y, Y*)
θ_i=2.0	(*Y, N*)	(*Y, N*)	(*Y, Y*)	(*Y, Y*)

θ_k is proportional to the average marginal cost in the specific functional form for a given throughput level. This example shows that if a port has a large average marginal cost, its expansion can effectively reduce congestion cost.

Example 5 *(Investment Cost) In this example, we set* $\bar{x}=1$, $\alpha_i=\alpha_j=0.5$, $\beta_i=\beta_j=0.1$, $\theta_i=\theta_j=1$, $C_i^0=C_j^0=2$, *and* $\Delta C=1$. *Table 7 provides equilibrium investment decisions for different values of the investment costs* I_i *and* I_j.

Table 7: Impacts of investment cost on the expansion decision

	I_j=0.010	I_j=0.012	I_j=0.014	I_j=0.016
I_i=0.010	(*Y, Y*)	(*Y, Y*)	(*Y, N*)	(*Y, N*)
I_i=0.012	(*Y, Y*)	(*Y, Y*)	(*Y, N*)	(*Y, N*)
I_i=0.014	(*N, Y*)	(*N, Y*)	(*N, N*)	(*N, N*)
I_i=0.016	(*N, Y*)	(*N, Y*)	(*N, N*)	(*N, N*)

Example 5 shows that the increase in investment cost can reduce the possibility for expansion. When two ports have different investment costs, the one with lower investment cost is more likely to expand.

4. A Case Study

In this section, we use numerical analysis to demonstrate the application of the model in analyzing the competition between Hong Kong and Shenzhen container ports, the two ports sharing the same hinterland. Compared with Shenzhen, Hong Kong port has a relatively larger market share, a larger initial capacity size, and a higher operation cost for the same throughput. To compete, both ports can consider capacity expansion and adjustment in price. For equal size capacity expansion, the container port in Hong Kong needs a higher investment cost. Therefore, in this numerical analysis, we use a smaller port with lower market share, lower operational and investment costs to represent Shenzhen, and the other port for Hong Kong, i.e., $\alpha_H>\alpha_S$, $\theta_H>\theta_S$, $I_H>I_S$, $C_H^0>C_S^0$, where the index H and S refer to Hong Kong and Shenzhen respectively. Let $\alpha_H=0.6$, α_S=0.4, $\theta_H=1.5$, θ_S=1.0, $I_H=0.05$, I_S=0.03, $C_H^0=2.0$, C_S^0,=1.5, ΔC=1. In Table 8 and 9, we calculate the Nash equilibrium in expansion strategy, equilibrium prices (p_H^*, p_S^*), and equilibrium profits ($\tilde{\Pi}_H^*$, $\tilde{\Pi}_S^*$) for different values of β_H and β_S when both ports compete using pricing and investment. The letters in the parenthesis stand for the strategies for Hong Kong port and Shenzhen port respectively.

Table 8: β_H=β_S=0.1

	$\bar{x}$ =1.2	$\bar{x}$ =1.4	$\bar{x}$ =1.6
Both ports do not invest	p_H^* =7.29 p_S^* =6.42 $\tilde{\Pi}_H^*$ =4.3142 $\tilde{\Pi}_S^*$ =3.4231	p_H^* =8.50 p_S^* =7.49 $\tilde{\Pi}_H^*$ =5.8722 $\tilde{\Pi}_S^*$ =4.6593	p_H^* =9.71 p_S^* =8.56 $\tilde{\Pi}_H^*$ =7.6698 $\tilde{\Pi}_S^*$ =6.0856
At investment equilibrium	(N, N) p_H^* =7.29 p_S^* =6.42 $\tilde{\Pi}_H^*$ =4.3142 $\tilde{\Pi}_S^*$ =3.4231	(N, Y) p_H^* =8.37 p_S^* =7.26 $\tilde{\Pi}_H^*$ =5.7000 $\tilde{\Pi}_S^*$ =4.6941	(Y, Y) p_H^* =9.30 p_S^* =8.15 $\tilde{\Pi}_H^*$ =7.5023 $\tilde{\Pi}_S^*$ =5.9238

Table 9: β_H=0.1, β_S=0.3

	$\bar{x}$ =1.2	$\bar{x}$ =1.4	$\bar{x}$ =1.6
Both ports do not invest	p_H^* =7.77 p_S^* =2.45 $\tilde{\Pi}_H^*$ =4.9133 $\tilde{\Pi}_S^*$ =1.0981	p_H^* =9.07 p_S^* =2.85 $\tilde{\Pi}_H^*$ =6.6876 $\tilde{\Pi}_S^*$ =1.4946	p_H^* =10.37 p_S^* =3.26 $\tilde{\Pi}_H^*$ =8.7348 $\tilde{\Pi}_S^*$ =1.9521
At investment equilibrium	(N, N) p_H^* =7.77 p_S^* =2.45 $\tilde{\Pi}_H^*$ =4.9133 $\tilde{\Pi}_S^*$ =1.0981	(N, Y) p_H^* =8.74 p_S^* =2.65 $\tilde{\Pi}_H^*$ =6.2073 $\tilde{\Pi}_S^*$ =1.5297	(Y, Y) p_H^* =9.70 p_S^* =2.97 $\tilde{\Pi}_H^*$ =8.1578 $\tilde{\Pi}_S^*$ =1.9276

There are several interesting results from these two tables.

1. Both ports will expand if and only if the base demand is high. This is illustrated in the last column ($\bar{x}$ =1.6) in both tables.
2. Compared with Shenzhen port, Hong Kong requires a larger base demand to justify the expansion decision. When $\bar{x}$ =1.4, the optimal strategies for Shenzhen (Hong Kong) in both table 8 and 9 are to expand (not to expand). This implies that Shenzhen port enjoys an advantageous position in capacity expansion.
3. Any port expansion will decrease the equilibrium prices at both ports. From both tables, whenever the equilibrium investment strategies include expansion at any port, the prices at both ports are lower than their corresponding prices with no expansion.
4. Shenzhen port can gain from expansion. For example, in Table 9, when $\bar{x}$=1.4, expansion in Shenzhen can increase its equilibrium profit from $\tilde{\Pi}_S^*$=1.4946 to $\tilde{\Pi}_S^*$=1.5297, a gain of 0.0351. On other hand, Hong Kong will always suffer a profit loss from expansion.
5. Furthermore, port expansion will not increase total profit of the two ports. For example, while Shenzhen port gains from expansion, it is less than the losses in Hong Kong. The total profit of the two ports is lower comparing with the non-expansion case. In Table 8, when $\bar{x} = 1.4$, Shenzhen can gain profit from expansion (4.6941-4.6593=0.0348). However, the total profit of the two ports decreased by (5.8722+4.6593)-(5.7000+ 4.6941)=0.1374. When the pure strategy Nash equilibrium is both expands, they all have a lower profit than that with no expansion, which resemble the Prisoner dilemma situation.

5. Summary

This paper studied the competition between two ports, serving the same hinterland, using both capacity expansion and pricing. Quantity demanded at each port is a function of its own and the competitor's prices. In addition to the short-run market competition measures, such as pricing, the increasing market demand from the same hinterland also provides opportunity for capacity expansion. For given throughput, port expansion can reduce the marginal operation cost, which can lead to lower user cost and higher market share. To counter balance this impact, the other port also needs to reduce its price(s), to maximize its overall profit.

Based on the decision making process for port expansion and pricing, this paper constructed a two-stage game theory model to analyze the possible outcomes in this duopoly market. Using backward induction, we first analyzed the pricing strategy of the two ports for given capacity at both ports. We showed the unique Nash equilibrium for the pricing subgame following the Bertrand competition with differentiated products. For each port, its equilibrium price increases with its marginal cost, base market demand, and market share. Port expansion can reduce prices at both ports, which is beneficial to the users. We also analyzed the impact on equilibrium throughput and profit with the capacity change.

In capacity investment game, we identified the pure strategy Nash equilibriums between two ports, for different scenarios characterized by the possible gains from port expansion, and the investment costs. Using numerical examples, we show that each port will be more inclined to expand when the total market demand is high, or it has a large market share. A port will be more likely to expand if it has high operational cost, low investment cost and low own price sensitivity. In considering expansion, for one unit decrease in price, if the difference in marginal cost increases between the expansion port and competitor is less than one, then the expansion can bring positive gain to the expanding port.

Our case study, based on numerical examples, demonstrates possible outcomes from port competition between Hong Kong and Shenzhen. If expansion is constrained to be one at a time, the numerical results show that both ports can expand only when the market demand is sufficiently high. Shenzhen is more likely to expand when the market is increasing, but not sufficient for both to expand. Port expansion can bring benefit to users, but not necessarily to ports. Shenzhen can benefit from its expansion if Hong Kong does not expand. However, the gain from expansion at Shenzhen port cannot compensate the losses at Hong Kong Port. If both expand, then each port will have a lower profit than if both do not expand.

Finally, this study focuses on the payoffs for the two ports from private business operator's perspective. The social outcomes of port competition with pricing and capacity development will be a direction for future study.

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Appendices

1. Comparative statics for the equilibrium price and profit.

Solving the two best response functions contained in (5) for the equilibrium price:
$p_k^* = \bar{x}(1+2f_k(C_k)\beta_k)\mu_k/(\lambda\beta_k)$,
where $\lambda = 3+2f_k(C_k)\beta_k + 2f_l(C_l)\beta_l$, and $\mu_k = 1+\alpha_k + 2f_l(C_l)\beta_l$

Differentiate the equilibrium price with respect to base market share, own price sensitivity and cross price sensitivity, we can obtain:

$$\frac{\partial p_k^*}{\partial \bar{x}} = (1+2f_k(C_k)\beta_k)\mu_k/(\lambda\beta_k) > 0$$

$$\frac{\partial p_k^*}{\partial \beta_k} = -\frac{\bar{x}\{3+4f_k(C_k)\beta_k[1+f_k(C_k)\beta_k]+2f_l(C_l)\beta_l\}\mu_k}{\lambda^2\beta_k^2} < 0$$

$$\frac{\partial p_k^*}{\partial \beta_l} = \frac{2\bar{x}f_l(C_l)[1+2f_k(C_k)\beta_k]\mu_l}{\beta_k\lambda^2} > 0$$

$$\frac{\partial p_k^*}{\partial \alpha_k} = \frac{2\bar{x}[1+2f_k(C_k)\beta_k][1+f_l(C_l)\beta_l]}{\beta_k\lambda} > 0$$

$$\frac{\partial p_k^*}{\partial \alpha_j} = \frac{\bar{x}[1+2f_k(C_k)\beta_k][1+2f_l(C_l)\beta_l]}{\beta_k\lambda} > 0$$

$$\frac{\partial p_k^*}{\partial C_k} = \frac{4\bar{x}[1+f_l(C_l)\beta_l]\mu_k f_k'(C_k)}{\lambda^2} < 0$$

$$\frac{\partial p_k^*}{\partial C_l} = \frac{2\bar{x}[1+2f_k(C_k)\beta_k]\mu_l\beta_l f_l'(C_l)}{\lambda^2\beta_k} < 0$$

From the demand function, we can obtain quantity demanded at the equilibrium by substituting the price with the equilibrium prices. The equilibrium quantity demanded at the equilibrium price is:

$$x_k^* = \frac{\bar{x}(1+\alpha_k+2f_l(C_l)\beta_l)}{3+2f_k(C_k)\beta_k+2f_l(C_l)\beta_l} = \frac{\bar{x}\mu_k}{\lambda}$$

The properties of the equilibrium throughput w.r.t. the demand parameters and capacity change are:

$$\frac{\partial x_k^*}{\partial \bar{x}} = \frac{\mu_k}{\lambda} > 0$$

$$\frac{\partial x_k^*}{\partial \alpha_k} = \frac{\bar{x}}{\lambda} > 0$$

$$\frac{\partial x_k^*}{\partial \alpha_l} = -\frac{\bar{x}}{\lambda} < 0$$

$$\frac{\partial x_k^*}{\partial \beta_k} = -\frac{2\bar{x}f_k(C_k)\mu_k}{\lambda^2} < 0$$

$$\frac{\partial x_k^*}{\partial \beta_l} = \frac{2\bar{x}f_l(C_l)\mu_l}{\lambda^2} > 0$$

$$\frac{\partial x_k^*}{\partial C_k} = -\frac{2\bar{x}\beta_k\mu_k f_k'(C_k)}{\lambda^2} > 0$$

$$\frac{\partial x_k^*}{\partial C_l} = \frac{2\bar{x}\beta_l[1+\alpha_k + 2f_k(C_k)\beta_k]f_l'(C_l)}{\lambda^2} < 0$$

Using the equilibrium price and quantity, we can calculate the equilibrium profit. Differentiate the equilibrium profit with respect to its own capacity and the capacity of the other port, we obtain:

$$\frac{\partial \pi_k^*}{\partial C_k} = -\frac{\bar{x}^2[1+2f_k(C_k)\beta_k - 2f_l(C_l)\beta_l]\mu_k f_k'(C_k)}{\lambda^3}$$

$$\frac{\partial \pi_k^*}{\partial C_l} = \frac{4\bar{x}^2[1+2f_k(C_k)\beta_k]\mu_l\beta_l\{2\alpha_k[1+f_l(C_l)\beta_l]\}f_l'(C_l)}{\beta_k\lambda^3}$$

Since $f_k'(C_k) < 0$, we can see that if $1+2f_k(C_k)\beta_k - 2f_l(C_l)\beta_l > 0$, $\frac{\partial \pi_k^*}{\partial C_k} > 0$; $\frac{\partial \pi_k^*}{\partial C_l}$ is always negative.

2. Proof of $\Delta\Pi_i^*(C_j) \geq 0$

$$\Delta\Pi_i^*(C_j) = \Pi_i^*(C_i^1, C_j) - \Pi_i^*(C_i^0, C_j)$$

$$\Pi_i^*(C_i^1, C_j) = \frac{\bar{x}[1+f_i(C_i^1)\beta_i][1+\alpha_i + 2\beta_j f_j(C_j)]^2}{\beta_i[3+2f_i(C_i^1)\beta_i + 2f_j(C_j)\beta_j]^2} \quad \text{(A-1)}$$

$$\Pi_i^*(C_i^0, C_j) = \frac{\bar{x}[1+f_i(C_i^0)\beta_i][1+\alpha_i + 2\beta_j f_j(C_j)]^2}{\beta_i[3+2f_i(C_i^0)\beta_i + 2f_j(C_j)\beta_j]^2} \quad \text{(A-2)}$$

$$\Delta\Pi_i^*(C_j) = \left\{\bar{x}[1+\alpha_i + 2\beta_j f_j(C_j)]^2\left[\frac{1+f_i(C_i^1)\beta_i}{[3+2f_i(C_i^1)\beta_i + 2f_j(C_j)\beta_j]^2} - \frac{1+f_i(C_i^0)\beta_i}{[3+2f_i(C_i^0)\beta_i + 2f_j(C_j)\beta_j]^2}\right]\right\} \Big/ \beta_i$$

The sign of above term is determined by the sign of the term in square bracket

$$\frac{1+f_i(C_i^1)\beta_i}{[3+2f_i(C_i^1)\beta_i + 2f_j(C_j)\beta_j]^2} - \frac{1+f_i(C_i^0)\beta_i}{[3+2f_i(C_i^0)\beta_i + 2f_j(C_j)\beta_j]^2} =$$

$$= \frac{1}{[3+2f_i(C_i^1)\beta_i + 2f_j(C_j)\beta_j]^2} - \frac{1}{[3+2f_i(C_i^0)\beta_i + 2f_j(C_j)\beta_j]^2} \quad \text{(Part A)}$$

$$+ \frac{f_i(C_i^1)\beta_i}{[3+2f_i(C_i^1)\beta_i + 2f_j(C_j)\beta_j]^2} - \frac{f_i(C_i^0)\beta_i}{[3+2f_i(C_i^0)\beta_i + 2f_j(C_j)\beta_j]^2} \quad \text{(Part B)}$$

$$\text{Part A} = \frac{[3+2f_i(C_i^0)\beta_i + 2f_j(C_j)\beta_j]^2 - [3+2f_i(C_i^1)\beta_i + 2f_j(C_j)\beta_j]^2}{[3+2f_i(C_i^1)\beta_i + 2f_j(C_j)\beta_j]^2[3+2f_i(C_i^0)\beta_i + 2f_j(C_j)\beta_j]^2}$$

$$= \frac{4\beta_i[f_i(C_i^0) - f_i(C_i^1)]\{3+\beta_i[f_i(C_i^0) + f_i(C_i^1)] + 2f_j(C_j)\beta_j\}}{[3+2f_i(C_i^1)\beta_i + 2f_j(C_j)\beta_j]^2[3+2f_i(C_i^0)\beta_i + 2f_j(C_j)\beta_j]^2}$$

$$\text{Part B} = \frac{4\beta_i^3 f_i(C_i^0)f_i(C_i^1)[f_i(C_i^0) - f_i(C_i^1)] - \beta_i[f_i(C_i^0) - f_i(C_i^1)][3+2f_j(C_j)\beta_j]^2}{[3+2f_i(C_i^1)\beta_i + 2f_j(C_j)\beta_j]^2[3+2f_i(C_i^0)\beta_i + 2f_j(C_j)\beta_j]^2}$$

Add two parts together, and factor out the common part,
Part A + Part B=

$$\frac{\beta_i\,[f_i(C_i^0) - f_i(C_i^1)]}{[3+2f_i(C_i^1)\beta_i + 2f_j(C_j)\beta_j]^2[3+2f_i(C_i^0)\beta_i + 2f_j(C_j)\beta_j]^2}$$

$$\times\left(4\{3+\beta_i[f(C_i^0)+f(C_i^1)]+2f(C_j)\beta_j\}+4f(C_i^0)f(C_i^1)\beta_i^2-[3+2f(C_j)\beta_j]^2\right)$$ --- part C

Part C= $\left(4[3+2f(C_j)\beta_j]+4\beta_i[f(C_i^0)+f(C_i^1)]+4f(C_i^0)f(C_i^1)\beta_i^2-[3+2f(C_j)\beta_j]^2\right)$

$=\left([3+2f(C_j)\beta_j][1-2f(C_j)\beta_j]+4\beta_i[f(C_i^0)+f(C_i^1)]+4f(C_i^0)f(C_i^1)\beta_i^2\right)$

$=\left(3-4f(C_j)\beta_j-4[f(C_j)\beta_j]^2+4\beta_i[f(C_i^0)+f(C_i^1)]+4f(C_i^0)f(C_i^1)\beta_i^2\right)$

The assumption ensures that $2\beta_i f_i(C^u)-2\beta_j f_j(C^l)+1>0$. Use this inequality and note $f_k(\cdot), k=i,j$ are decreasing functions, it can be proved that

$$\begin{aligned}
&3+4\beta_i f_i(C_i^0)+4\beta_i f_i(C_i^1)+4\beta_i^2 f_i(C_i^1)f_i(C_i^0)-4\beta_j f_j(C_j)-4\beta_j^2 f_j^2(C_j)\\
&>3+4\beta_i f_i(C^u)+4\beta_i f_i(C^u)+4\beta_i^2 f_i^2(C^u)-4\beta_j f_j(C^l)-4\beta_j^2 f_j^2(C_j)\\
&>1+4\beta_i f_i(C^u)+[2\beta_i f_i(C^u)-2\beta_j f_j(C^l)][2\beta_i f_i(C^u)+2\beta_j f_j(C^l)]\\
&>1+4\beta_i f_i(C^u)-[2\beta_i f_i(C^u)+2\beta_j f_j(C^l)]\\
&=1+2\beta_i f_i(C^u)-2\beta_j f_j(C^l)\\
&>0.
\end{aligned}$$

Therefore, $\Delta\Pi_i^*(C_j)\geq 0$.