

# The effect of risk aversion on manufacturer promotion in a two-stage supply chain

Donghan Liang<sup>1a</sup>, Gang Li<sup>1b</sup>, Jie Gao<sup>1c</sup>, Linyan Sun<sup>1d</sup> and Xinyu Sun<sup>2</sup>

<sup>1</sup>The Management School of Xi'an Jiaotong University, The State Key Lab for Manufacturing Systems Engineering, Xi'an, 710049, China.

<sup>a</sup>E-mail: dhsweet@yahoo.com.cn

<sup>b</sup>E-mail: glee@mail.xjtu.edu.cn

<sup>c</sup>E-mail: gaoj@mail.xjtu.edu.cn

<sup>d</sup>E-mail: lysun@mail.xjtu.edu.cn

<sup>2</sup>Department of Logistics and Maritime Studies, The Hong Kong Polytechnic University, Hong Kong.  
E-mail: lgtxysun@inet.polyu.edu.hk

## Abstract

We consider a supply chain system with a risk-neutral manufacturer as the leader and a risk-averse retailer as the follower in the environment with uncertain demand. At the beginning of the game, the manufacturer makes investment on promotion effort and then the retailer decides his ordering quantity before demand realization. The analysis of equilibrium strategies of this Stackelberg game indicates some characteristics which are different from promotion strategies with risk neutral agents. Firstly, there exists an upper bound for the retailer's target profit  $\alpha$ , otherwise the equilibrium strategy is unavailable. Secondly, the retailer's risk aversion has direct influence on the manufacturer's promotion investment. In other words, the manufacturer will increase his promotional effort when the retailer has an appropriate degree of risk aversion and cuts down that for a highly risk-averse one. Thirdly, although conventional wisdom suggests that risk-averse retailer definitely reduce his ordering quantity, we find that manufacturer's promotion can effectively prevent the risk-averse retailer from downsizing inventory which is decided by the joint power of the promotion effort  $\rho$  and the variable pair  $(\alpha, \beta)$ .

*Key word:* Risk aversion; Promotion effort; Newsvendor

## 1. Introduction

The power of promotion has been identified in abundant literatures on marketing as well as supply chain management. Promotional activities are implemented by both suppliers and retailers, spreading from media advertising, sponsoring for business events to sending catalogues and salesmen's effort. Typically, promotion activities are simplified to advertising for model analysis and classified into brand advertising and local advertising according to the agent responsible for the matter. Generally speaking, brand advertising is always implemented by the brand owner, widely known as the supplier or manufacturer, to make his product differentiated. In other words, the manufacturer expects to grab potential demand and to develop brand knowledge and preference through brand advertising (Huang and Li, 2001). However, these days, as retailers are getting dominant in supply chain partnerships, famous retailers, such as Wal-mart, Go-me, Suning etc., also make brand advertising for their own good, which is interesting but beyond the range of this paper.

Most marketing studies to date on promotion or advertising have focused on how customers respond to retailers' sales effort, that is, marketing researches mainly concern retailer's local advertising or promotion and the relationship between retailers and consumers. On the other side, in the field of supply chain management, literatures on vertical co-op advertising constitutes mainstream of advertising studies, which typically consider the decision making process arises between

manufacturers and retailers and the equilibrium optimizing the whole supply chain performance. Basically, these papers discuss advertising in general terms, that is, the members involved are assumed identical in behavioral terms such as rational hypothesis, risk attitude etc.. Actually, the way advertising as well as other promotional activities works closely relates to these behavioral features that impact on the players' strategy choice.

Proved to be influential to decision making, risk attitude is a dominant feature of supply chain members and plays an important role in supply chain performance. Most recent studies on optimal stock level take the buyer's risk attitude into account, especially when the buyer is risk averse, to acquire meaningful insights to improve supply chain efficiency. Early risk aversion measures include utility maximization and mean-variance analysis, still in use and illustrative for a number of scenarios. Recently, the introduction of financial measures, the so-called VaR and CVaR, bridging supply chain members' risk aversion with his target profit directly, has greatly enriched the meaning of risk aversion and made analysis more applicable. Likewise, studies with VaR or CVaR approaches on supply chain management also mainly focus on the inventory issue, while the interaction between advertising or promotional activities and risk-averse members is lack of investigation.

This paper follows Gan's (2005) work on supply chain coordination with a risk-averse retailer, in which the concept of downside risk is introduced and employed to illustrate the retailer's inventory strategy and the supplier's coordination contract. The present paper follows most of Gan's (2005) assumptions and takes the view of promotion to investigate supply chain members' equilibrium strategies. Targeting at demonstrating the impact of brand advertising on the retailer's decision making, a Stackelberg game is employed where the supplier act as the leader and the retailer the follower. We find that the risk-neutral manufacturer increases promotional effort in a certain range determined by the retailer's risk-aversion degree and the risk-averse retailer does not necessarily order less than that of a risk-neutral one.

The paper is organized as follows: in section 2 we review literatures concerned; in section 3 we propose our assumptions, model a Stackelberg game with downside risk constraints and report our insights; in section 4 we make the conclusion and discussion part.

## **2. Literature Review**

Conventional wisdom suggests advertising a powerful promotional tool widely investigated in both marketing and SCM criteria. One branch of marketing studies attaches more importance to the advertising attributes such as information, content, formality etc., (e.g. Bloch and Manceau, 1999; Dukes and Gal-Or, 2003); the other branch considers advertising as a principle marketing strategy (e.g. Narasimhan et al. 1996; Drèze and Bell 2002). It is the marketing perspective on advertising that accounts for the customer-oriented research schemes in which the relationship between supply chain members does not concern much, but the insights it provided are meaningful to brand promotion and market attracting for enterprise competition.

On the contrary, SCM papers with promotion scenarios are highly interested in its influence on supply chain members' relationship on cost and profit sharing, which is widely investigated in the criteria of retailer promotion and co-op vertical advertising recently. Cachon (2002) thoroughly reviewed supply chain coordination studies on newsvendor with demand dependent on the retailer's promotional effort and summarized needed conditions under which the supplier would share the retailer's promotion expense and the supply chain can be coordinated. He emphasized that the promotional cost should be observable to the supplier and verifiable to the court and directly benefits the supplier otherwise the cost sharing contract cannot be implemented. Constrained by the rule mentioned above, most studies on promotion either particularly demonstrate the definition of promotional effort in their model or directly choose advertising as promotional parameters for its observable and verifiable cost structure. Generally, the retailer's closeness to end consumers facilitates various promotional activities which have been widely investigated as a significant supply chain phenomenon. Wang and Gerchak (2001) assumed the demand for a certain product is influenced by its display level controlled by the retailer and indicated that the manufacturer should compensate the retailer with an extra holding cost to

coordinate the channel and make a profit. Taylor (2002) proposed a supply chain coordination contract for which the retailer has to decide his promotional effort in addition to inventory level. Krishnan et al (2004) extended Taylor's (2002) research and investigated the coordination mechanism when the retailer chooses inventories ex ante and promotional effort ex post. In fact, what the above papers concerns is appropriate compensation mechanism for the retailer who bears the promotional cost beneficial to the supplier as well. The contracts designed for this purpose discreetly split the cost and profit to insure the compensation would reduce the retailer's risk without compromising his effort degree. Although risk attitude was excluded in these studies, they performed as risk-sharing tools in common. Suo et al (2005) explicitly presented a model considering the impacts of the retailer's loss-aversion on his promotional effort. Using a loss-aversion utility function, they found that loss aversion weakens incentives for retailer's sales effort.

Literatures on retailer's promotion mainly establish analysis on the basis of newsvendor problem, while studies on vertical co-op advertising usually develop models with deterministic demand denoted by a function of retailing price and channel members' advertising investment. Jorgensen and Zaccour (1999) proposed a differential game to study channel coordination and channel conflict with channel members' advertising and pricing strategies, proved the existing of closed-form solutions for both scenarios and obtained a global result that the cooperative scenario supports greater level of adverting investments from both members. Thereafter studies on co-op advertising mainly concerns advertisement efforts in dimensions such as national level expenditures, local level expenditures, manufacturer participation rate, sales volume, and brand and store substitutions (Xie and Wei, 2009). Huang and Li (2001) developed three co-op advertising models to explain the cost-sharing issue between the manufacturer and the retailer. For the cooperative model they originally employed Nash bargaining game and took supply chain members' risk attitude into account. Utilizing the Pratt-Arrow risk aversion function they found that the manufacturer shares less of the local advertising cost if the retailer has a higher degree of risk aversion.

Literatures reviewed above typically consider supplier advertising as a supplement to retailer's promotion even with retailer's risk aversion involved. This paper develops a model with a risk-averse retailer and upstream promotion and illustrates the significant role played by supplier promotion in the sense of risk sharing.

### 3. Model

We now consider a Stackelberg game that consists of a risk-neutral manufacturer  $M$  and a risk-averse retailer  $R$ , in which the manufacturer  $M$  performs as the leader and the retailer plays as the follower. Based on the newsvendor, we suppose the transaction contains a single perishable product with a random market demand  $X$  (i.e. the deterministic quantity of  $X$  can not be observed before the selling season) that has a distribution density  $f(x)$  and distribution function  $F(x)$  known as common knowledge to both the manufacture and the retailer. The two players moves in following sequence: first the manufacturer promotes his product with effort  $\rho$  to enlarging the market demand at an expense  $V(\rho)$ , increasing on  $\rho$  with  $V'(\rho) \geq 0, V''(\rho) \geq 0$ , extending the original demand  $X$  to  $\rho X$  which realizes when the selling season begins; then the manufacture wholesales products to the retailer at unit cost  $c$  and receive  $w$  each unit, and the retailer will sell them on the market at price  $p$  per unit; finally the enlarged market demand  $\rho X$  is observed. For the simplicity of our analysis, we assume the goodwill cost and salvage value of the perishable product is zero for both the manufacturer and the retailer. We also assume that each player targets at optimizing his expected profit within the constraint and there's no information asymmetry.

There are two critical decision variables in the system above-the manufacturer's promotion effort  $\rho$  and the retailer's ordering quantity  $q$ -taking up our main concern in following analysis. The above introduction of our model mostly inherits the traditional newsvendor with promotion problem and the retailer's downside risk constraint is presented in this part. The concept of downside risk was employed by Gan (2005), implying a bearable biggest failure rate describing the probability that the

agent can not achieve his target profit, thereby the retailer would reduce his ordering quantity as long as the downside risk exceed his limitation. The constraint is also constructed accordingly: suppose the retailer has a target profit  $\alpha$  and downside risk  $\beta$  and his risk constraint is written as:

$$P(\Pi_r \leq \alpha) \leq \beta \quad (1)$$

In which  $\Pi_r$  represents the retailer's profit and  $\Pi_r = p \min(q, \rho X) - wq$ .

Expected profit functions for the manufacturer, the retailer and the system are determined by following equations:

$$\pi_m = (w - c)q - V(\rho) \quad (2)$$

$$\pi_r = pE \min(q, \rho X) - wq \quad (3)$$

$$s.t. P(\Pi_r \leq \alpha) \leq \beta$$

$$\pi_s = \pi_m + \pi_r = pE \min(q, \rho X) - cq - V(\rho) \quad (4)$$

Then we solve for the non-cooperative sequential game with the manufacturer as the leader and the retailer as the follower and the result is Stackelberg equilibrium.

We begin with the retailer's ordering strategy considering his downside risk constraint. Let  $q^*$  be the optimal ordering quantity of the retailer whose maximization problem is described in programming (3) and the objective function is  $\max \pi_r$ . We consider the risk constraint best of all and split it into two scenarios:  $q \leq \rho X$  and  $q > \rho X$  (in which the variable  $\rho$  is treated as a known constant because it has been decided by the manufacturer at the first stage of the game). For the first scenario, all the products are sold out and constraint (1) is equal to the expression below:

$$P((p - w)q \leq \alpha) \leq \beta \quad (5)$$

So we get the lower bound of the retailer's optimal order quantity  $q^0 = \frac{\alpha}{p - w}$ . The retailer make a

profit no more than  $(p - w)q$  given his ordering quantity  $q$ . Therefore, if the order quantity is less than  $q^0$ , the target profit  $\alpha$  can never be achieved and the retailer has to order more than  $q^0$  to meet his target profit. When  $q^0 < q \leq \rho X$ , the retailer would deterministically gain more than  $\alpha$  and the downside risk is zero, therefore, the constraint binds only if  $q > q^0$  and  $q > \rho X$ . For the second scenario, we have

$$P(p\rho X - wq \leq \alpha) = P(X \leq \frac{\alpha + wq}{p\rho}) = F(\frac{\alpha + wq}{p\rho}) \leq \beta \quad (6)$$

Expression (6) relates the demand distribution function  $F(x)$  to the downside risk  $\beta$ , which is critical to our further analysis. With some manipulation on expression (6) we get an upper bound for  $q$ :  $q \leq \frac{p\rho F^{-1}(\beta) - \alpha}{w}$  when the constraint binds. Let  $\rho^*$  be the optimal promotion effort invested by the manufacturer in the first stage and  $(\bar{\rho}, \bar{q})$  be the equilibrium strategy for traditional newsvendor (all the players are risk neutral). Let  $(\rho_h^*, q_h^*)$  be the equilibrium strategy when  $\beta \geq F(\frac{\alpha + w\bar{q}}{p\bar{\rho}})$  and  $(\rho_l^*, q_l^*)$  when  $F(q^0) < \beta < F(\frac{\alpha + w\bar{q}}{p\bar{\rho}})$ . Theorem 1 describes the equilibrium strategy  $(\rho^*, q^*)$  with dynamic  $\alpha$  and  $\beta$  value.

### Theorem 1:

① If  $0 < \alpha \leq \frac{\bar{\rho}}{\bar{\rho} - (1 - \frac{w}{p})} \cdot \frac{w}{p} (p - w) F^{-1}(\frac{p - w}{p})$ , then

(1) when  $\beta \geq F(\frac{\alpha + w\bar{q}}{p\bar{\rho}})$ ,  $q_h^* = \bar{q} = \bar{\rho} F^{-1}(\frac{p - w}{p})$ ,  $\rho_h^* = \bar{\rho}$ ,  $V'(\rho)|_{\rho=\bar{\rho}} = (w - c) F^{-1}(\frac{p - w}{p})$ ;

(2) when  $F(q^0) < \beta < F(\frac{\alpha + w\bar{q}}{p\bar{\rho}})$ ,  $q_i^* = \frac{p\rho_i^* F^{-1}(\beta) - \alpha}{w}$ ,  $V'(\rho)|_{\rho=\rho_i^*} = \frac{(w - c)p}{w} F^{-1}(\beta)$

(3) when  $\beta \leq F(q^0)$ , there is no equilibrium solution.

② If  $\frac{\bar{\rho}}{\bar{\rho} - (1 - \frac{w}{p})} \cdot \frac{w}{p} (p - w) F^{-1}(\frac{p - w}{p}) < \alpha \leq \bar{\rho}(p - w) F^{-1}(\frac{p - w}{p})$ , then the equilibrium strategy

is  $(\bar{\rho}, \bar{q})$  with any  $\beta$  that satisfies  $F(q^0) < \beta < 1$ .

③ If  $\alpha > \bar{\rho}(p - w) F^{-1}(\frac{p - w}{p})$ , there is no available solution.

**Proof:**

① If  $0 < \alpha \leq \frac{\bar{\rho}}{\bar{\rho} - (1 - \frac{w}{p})} \cdot \frac{w}{p} (p - w) F^{-1}(\frac{p - w}{p})$ , then we have  $F(q^0) \leq F(\frac{\alpha + w\bar{q}}{p\bar{\rho}})$ .

(1) When  $\beta \geq F(\frac{\alpha + w\bar{q}}{p\bar{\rho}})$ , the retailer's downside risk constraint does not bind, therefore the retailer's order decision is the same as that of the traditional newsvendor, which is given by  $\bar{q} = \rho F^{-1}(\frac{p - w}{p})$ .

Then we substitute  $\bar{q}$  into Eq. (2) and solve  $\max_{\rho \geq 1} \pi_m$  for the manufacturer's optimal promotional effort  $\bar{\rho}$ , which can be simply obtained through first derivative condition. The equilibrium strategy  $(\bar{\rho}, \bar{q})$  takes the form

$$V'(\rho)|_{\rho=\bar{\rho}} = (w - c) F^{-1}(\frac{p - w}{p}), \quad \bar{q} = \bar{\rho} F^{-1}(\frac{p - w}{p}).$$

(2) when  $F(q^0) < \beta < F(\frac{\alpha + w\bar{q}}{p\bar{\rho}})$ , the downside risk constraint binds and the retailer's maximization problem becomes

$$\begin{aligned} & \max_{q \geq 0} \pi_r \\ & s.t. \quad P(\Pi_r \leq \alpha) = \beta \end{aligned} \tag{*}$$

We can now derive the solution to programming (\*), as shown below:

$$q^* = \frac{p\rho F^{-1}(\beta) - \alpha}{w} \tag{7}$$

On substituting this equation into Eq. (2) and solving for  $\rho^*$ , we obtain  $(\rho^*, q^*)$  as follows:

$$V'(\rho)|_{\rho=\rho^*} = \frac{(w - c)p}{w} F^{-1}(\beta), \quad q^* = \frac{p\rho^* F^{-1}(\beta) - \alpha}{w}.$$

(3) When  $\beta \leq F(q^0)$ , there is no such  $q$  that achieve the target profit  $\alpha$ , making the whole problem unsolvable.

$$\textcircled{2} \text{ If } \frac{\bar{\rho}}{\bar{\rho} - (1 - \frac{w}{p})} \cdot \frac{w}{p} (p-w) F^{-1}(\frac{p-w}{p}) < \alpha \leq \bar{\rho} (p-w) F^{-1}(\frac{p-w}{p}),$$

then  $F(\frac{\alpha + w\bar{q}}{p\bar{\rho}}) < F(q^0) < F(\bar{q})$ . When  $\beta \leq F(q^0)$ , it is obvious that no appropriate  $q$  matches;

when  $F(q^0) < \beta \leq 1$ , it can be deduced that we have  $\beta > F(\frac{\alpha + w\bar{q}}{p\bar{\rho}})$ , so the downside risk constraint does not bind, and we obtain the equilibrium strategy  $(\bar{\rho}, \bar{q})$ .

$$\textcircled{3} \text{ If } \alpha > \bar{\rho} (p-w) F^{-1}(\frac{p-w}{p}), \text{ then } F(\frac{\alpha + w\bar{q}}{p\bar{\rho}}) < F(\bar{q}) < F(q^0). \text{ When } F(q^0) < \beta \leq 1, \beta > F(\frac{\alpha + w\bar{q}}{p\bar{\rho}}),$$

so the optimal order quantity is  $\bar{q}$ , which contradicts the fact  $\bar{q} < q^0$ , leaving our problem unsolvable.

Note that the retailer's target profit can not exceed his revenue in risk-neutral setting otherwise there is no appropriate ordering quantity that matches the downside risk constraint. Theorem 1 also shows that when the target profit below the risk-neutral revenue, higher target profit setting ( $\alpha$ ) requires bigger downside risk ( $\beta$ ) for available solutions. When the retailer is highly risk-averse with high target profit and low downside risk, it is almost impossible to give a deterministic estimation for equilibrium solution because of the lower bound constraint  $\beta > F(q^0)$ . It is interesting that a risk-averse retailer's ordering quantity is not necessarily lower than that of a risk-neutral one due to the impacts of the supplier's promotion effort, a sharp contrast to the situation without supplier's promotion effort in which the risk-averse retailer order strictly less than the risk-neutral one presented in Gan's (2005) work. We start with Theorem 2 on the investigation of the relationship between the two elements  $\rho^*$  and  $q^*$  of the Stackelberg equilibrium strategy and the comparison of them with different concerning parameters.

**Theorem2:** Considering  $0 < \alpha \leq \frac{\bar{\rho}}{\bar{\rho} - (1 - \frac{w}{p})} \cdot \frac{w}{p} (p-w) F^{-1}(\frac{p-w}{p})$  and  $\beta > F(q^0)$  only:

$$\textcircled{1} \text{ If } \frac{w}{p} (p-w) F^{-1}(\frac{p-w}{p}) < \alpha \leq \frac{\bar{\rho}}{\bar{\rho} - (1 - \frac{w}{p})} \cdot \frac{w}{p} (p-w) F^{-1}(\frac{p-w}{p}), \text{ then } \rho_{l_1}^* \geq \rho_h^* ;$$

$$\textcircled{2} \text{ If } 0 < \alpha \leq \frac{w}{p} (p-w) F^{-1}(\frac{p-w}{p}), \text{ then two scenarios are considered:}$$

$$(1) \text{ when } F(\frac{w}{p} F^{-1}(\frac{p-w}{p})) < \beta \leq F(\frac{\alpha + w\bar{q}}{p\bar{\rho}}), \text{ we have } \rho_{l_2}^* \geq \rho_h^* ;$$

$$(2) \text{ when } F(q^0) < \beta \leq F(\frac{w}{p} F^{-1}(\frac{p-w}{p})), \text{ we have } \rho_{l_3}^* \leq \rho_h^* .$$

**Proof:**

$$\textcircled{1} \text{ If } \frac{w}{p} (p-w) F^{-1}(\frac{p-w}{p}) < \alpha \leq \frac{\bar{\rho}}{\bar{\rho} - (1 - \frac{w}{p})} \cdot \frac{w}{p} (p-w) F^{-1}(\frac{p-w}{p}), \text{ obviously we}$$

$$\text{have } \frac{w}{p} F^{-1}(\frac{p-w}{p}) < \frac{\alpha}{p-w} \leq \frac{\bar{\rho}}{\bar{\rho} - (1 - \frac{w}{p})} \cdot \frac{w}{p} F^{-1}(\frac{p-w}{p}), \text{ which is in equivalence with the}$$

following expression:

$$F(\frac{w}{p} F^{-1}(\frac{p-w}{p})) < F(q^0) \leq F(\frac{\alpha + w\bar{q}}{p\bar{\rho}}) \quad (8)$$



With  $\beta > F(q^0)$  we can deduce that  $\beta > F(\frac{w}{p}F^{-1}(\frac{p-w}{p}))$  and  $F^{-1}(\beta) > \frac{w}{p}F^{-1}(\frac{p-w}{p})$ .

Given that  $V'(\rho)|_{\rho=\bar{p}}=(w-c)F^{-1}(\frac{p-w}{p})$  and  $V'(\rho)|_{\rho=\rho_l^*}=\frac{(w-c)p}{w}F^{-1}(\beta)$ , we compare  $\rho_l^*$  to  $\bar{p}$  and find that  $\rho_l^* \geq \bar{p}$ .

② If  $0 < \alpha \leq \frac{w}{p}(p-w)F^{-1}(\frac{p-w}{p})$ , then  $F(q^0) \leq F(\frac{w}{p}F^{-1}(\frac{p-w}{p})) < F(\frac{\alpha+w\bar{q}}{p\bar{p}})$ .

Therefore two ranges of  $\beta$  are considered with the downside risk constraint binding:

when  $F(\frac{w}{p}F^{-1}(\frac{p-w}{p})) < \beta \leq F(\frac{\alpha+w\bar{q}}{p\bar{p}})$ , we have  $V'(\rho_l^*) > V'(\rho_h^*)$  and  $\rho_l^* \geq \rho_h^*$ ;

when  $F(q^0) < \beta \leq F(\frac{w}{p}F^{-1}(\frac{p-w}{p}))$ , we have  $V'(\rho_l^*) \leq V'(\rho_h^*)$  and  $\rho_l^* \leq \rho_h^*$ .

Theorem 2 demonstrates the  $\rho$  value under different pairs of  $\alpha$  and  $\beta$ , where the manufacturer's willing-to-pay investment in promotion or advertising alters as the retailer's risk attitude changes. It is reasonable that when the retailer becomes increasingly risk-averse-seeking for higher target profit and lower downside risk-the manufacture improves on his promotion effort as a signal of enlarging demand to boost the retailer's ordering quantity. However, the manufacturer would not keep on increasing promotional expense forever: if the retailer is conservative on his downside risk and expects little revenue, mass investment in promotion is improper for the manufacturer and he would cut down advertising cost to avoid potential loss. Moreover, we find that no evidence suggests that the retailer with highest downside risk and lowest target profit ensures the manufacturer's best promotional investment, which implies that the risk-neutral manufacturer prefer moderate risk aversion rather than boldness when dealing with the retailer. The analysis above reveals that the retailer's risk aversion typically influences the manufacturer's promotion decision, however, the manufacturer's promotion ultimately targets at extending consumer demand so that the retailer does not hesitate to increase inventory. We can expect that the manufacturer's promotion decision has an impact on the retailer's ordering decision in reverse. Therefore, the retailer's ordering quantity is influenced by both his own risk attitude and the manufacture's promotion effort, making his inventory decision complicated. Theorem 3 investigates the retailer's inventory decision considering promotion effect.

**Theorem 3:**

① If  $\frac{w}{p}(p-w)F^{-1}(\frac{p-w}{p}) < \alpha \leq \frac{\bar{p}}{\bar{p}-(1-\frac{w}{p})} \cdot \frac{w}{p}(p-w)F^{-1}(\frac{p-w}{p})$  and  $\rho_l^* \geq \frac{\bar{p}^2}{\bar{p}-(1-\frac{w}{p})}$ ,

then  $q_l^* \geq q_h^*$ ;

② If  $0 < \alpha \leq \frac{w}{p}(p-w)F^{-1}(\frac{p-w}{p})$  and  $\rho_l^* \geq \frac{\bar{p}^2}{\bar{p}-(1-\frac{w}{p})}$ , when the range of  $\beta$  satisfies

$$F(\frac{w}{p}F^{-1}(\frac{p-w}{p})) < \beta \leq F(\frac{\alpha+w\bar{q}}{p\bar{p}}), \text{ then } q_l^* \geq q_h^* ;$$

③ If  $0 < \alpha \leq \frac{w}{p}(p-w)F^{-1}(\frac{p-w}{p})$  and  $F(q^0) < \beta \leq F(\frac{w}{p}F^{-1}(\frac{p-w}{p}))$ ,  $q_l^* \leq q_h^*$ .

**Proof:**

① If  $0 < \alpha \leq \frac{\bar{\rho}}{\bar{\rho} - (1 - \frac{w}{p})} \cdot \frac{w}{p} (p - w) F^{-1}(\frac{p - w}{p})$ , then we can deduce that

$$q_l^* - q_h^* \geq \frac{p}{w} \rho_l^* F^{-1}(\beta) - \frac{\bar{\rho}^2}{\bar{\rho} - (1 - \frac{w}{p})} F^{-1}(\frac{p - w}{p});$$

Suggested in Theorem 2, there are two scenarios in which  $F^{-1}(\beta) \geq \frac{w}{p} F^{-1}(\frac{p - w}{p})$ :

when  $\frac{w}{p} (p - w) F^{-1}(\frac{p - w}{p}) < \alpha \leq \frac{\bar{\rho}}{\bar{\rho} - (1 - \frac{w}{p})} \cdot \frac{w}{p} (p - w) F^{-1}(\frac{p - w}{p})$  or

when  $0 < \alpha \leq \frac{w}{p} (p - w) F^{-1}(\frac{p - w}{p})$  with  $F(\frac{w}{p} F^{-1}(\frac{p - w}{p})) < \beta \leq F(\frac{\alpha + w\bar{q}}{p\bar{\rho}})$ .

$$\rho_l^* \geq \frac{\bar{\rho}^2}{\bar{\rho} - (1 - \frac{w}{p})} \quad \rho_l^* F^{-1}(\beta) \geq \frac{w}{p} \frac{\bar{\rho}^2}{\bar{\rho} - (1 - \frac{w}{p})} F^{-1}(\frac{p - w}{p})$$

Therefore, for  $\frac{\bar{\rho}^2}{\bar{\rho} - (1 - \frac{w}{p})}$ , we have  $\rho_l^* F^{-1}(\beta) \geq \frac{w}{p} \frac{\bar{\rho}^2}{\bar{\rho} - (1 - \frac{w}{p})} F^{-1}(\frac{p - w}{p})$ , and the expression  $q_l^* - q_h^* \geq 0$  is proved, leading to  $q_l^* \geq q_h^*$ .

② If  $0 < \alpha \leq \frac{w}{p} (p - w) F^{-1}(\frac{p - w}{p})$  and  $F(q^0) < \beta \leq F(\frac{w}{p} F^{-1}(\frac{p - w}{p}))$ , we directly have

$$F^{-1}(\beta) \leq \frac{w}{p} F^{-1}(\frac{p - w}{p}), \text{ so that } \rho_l^* \leq \rho_h^* \text{ and } q_l^* - q_h^* \leq \frac{p}{w} \rho_l^* F^{-1}(\beta) - \bar{\rho} F^{-1}(\frac{p - w}{p}) \leq 0,$$

therefore  $q_l^* \leq q_h^*$  is proved.

It is clearly illustrated in Theorem 3 that manufacturer's promotion changes the conventional phenomenon that a risk-averse retailer always ordering less than a risk-neutral one. Certainly the manufacturer has to pay more on promotion as a motivation towards the retailer to boost sales. As a result, the risk-averse retailer is encouraged to buy more from the manufacturer. Note that even if the retailer orders more than a risk-neutral retailer, the manufacturer may not gain extra revenue considering his expense on promotion. This suggests that the manufacturer has to balance his promotion cost and wholesale revenue facing a risk-averse retailer.

According to Gan (2005), the retailer's risk-aversion can be classified with variable pairs  $(\alpha, \beta)$ , the combination of a higher  $\alpha$  and a lower  $\beta$  means increasing risk aversion. However, our analysis indicates that a pair of higher  $\alpha$  and lower  $\beta$  probably yields unmatched target profit and downside risk (see Theorem 1). Therefore we cannot advise on the situation that highly risk-averse retailer is involved, and we only concerns appropriate pair of  $(\alpha, \beta)$ .

#### 4. Conclusion

As discussed in Section 1, this paper is motivated by the desire to explain a common market phenomenon regularly neglected by researchers on marketing or SCM. To extend the scale of our work, we employ methods in different fields including marketing, SCM and finance. This is also similar to the environment where real firms are operated, with various elements impacting on each other. As for the model construction, we follow the form of downside risk analysis with Stackelberge game, that is, a combination of financial measure with game theory which seems to be sustainable in



the sense of approach.

Our main concerns are the relationship between supply chain members with different risk attitude when promotion is considered. To address this issue, we compare the equilibrium strategies under different scenarios; although no firm judgment is made, we do obtain some insights about the interaction of the retailer and the manufacturer. Firstly, there exists an upper bound for the retailer's target profit  $\alpha$ , otherwise the equilibrium strategy is unavailable. Secondly, the retailer's risk aversion has direct influence on the manufacturer's promotion investment. In other words, the manufacturer will increase his promotional effort when the retailer has an appropriate degree of risk aversion and cuts down that for a highly risk-averse one. Thirdly, although conventional wisdom suggests that risk-averse retailer definitely reduce his ordering quantity, we find that manufacturer's promotion can effectively prevent the risk-averse retailer from downsizing inventory which is decided by the joint power of the promotion effort  $\rho$  and the variable pair  $(\alpha, \beta)$ .

This work is an exploring job on researches about promotion and risk and there are still many unsolved problems, e.g. designing contracts to coordinate the supply chain and reallocate market risk, the introduction of competition mechanisms, etc.. Our research will go deep into this topic step by step to obtain more insights for both study and real world application.

## References

- Bloch, F. and Manceau, D., (1999), Persuasive advertising in Hotelling's model of product differentiation, *International Journal of Industrial Organization*, 17(4):557-574.
- Cachon, G. (2002), Supply chain coordination with contracts. <http://opim.wharton.upenn.edu/~cachon>.
- Drèze, X. and Bell, D.R., (2002), Changing the Channel: A Better Way to do Trade Promotions, *Sloan Management Review*, 43 (2): 42-49.
- Dukes, A. and Gal-Or, E., (2003), Minimum Differentiation in Commercial Media Markets, *Journal of Economics & Management Strategy*, 12(3): 291-325.
- Gan, X H., Sethi, s. and Yan, H.M.. (2005), Channel Coordination with a Risk-Neutral Supplier and a Downside-Risk-Averse Retailer, *Production and Operation Management* 14(1): 80-89.
- Huang, Z.M. and Li, S.X., (2001), Co-op advertising models in manufacturer-retailer supply chains: A game theory approach, *European Journal of Operational Research*, 135(3): 527-544.
- Jorgensen, S. and Zaccour, G., (1999), Equilibrium Pricing and Advertising Strategies in A Marketing Channel, *Journal of Optimization Theory and Applications*, 102 (1): 111-125.
- Krishnan, H., Kapuscinski, R. and Butz, D. A. (2004), Coordinating Contracts for Decentralized Supply Chains with Retailer Promotional Effort, *Management Science*, 50(1): 48-63.
- Narasimhan, C., Neslin, S. A. and Sen, S.K., (1996), Promotional Elasticity and Category Characteristics, *Journal of Marketing*, (60):17-30.
- Suo, H.S., Wang, J.C. and Jin, Y.H., (2005), Supply Chain Coordination with Sales Effort Effects and Impact of Loss Aversion on Effort Decision, *Tsinghua Science and Technology*, 10(1): 102-107
- Taylor, T. A. (2002), Supply chain coordination under channel rebates with sales effort effects, *Management Science*, 48(8): 992-1007.

Wang, Y. Z. and Gerchak, Y., (2001), Supply Chain Coordination When Demand is Shelf-space Dependent, *Manufacturing & Service Operations Management*, 3(1): 82-87.

Xie, J.X. and Wei, J. C., (2009), Coordinating advertising and pricing in a manufacturer–retailer channel, *European Journal of Operational Research*, 197(2): 785-791.